

1) Calcolare le seguenti composizioni.

$$\left. \begin{array}{l} f(x) = \sin x \\ g(x) = e^x \end{array} \right\} \rightarrow \begin{array}{l} (g \circ f)(x) \\ (f \circ g)(x) \end{array} \quad \left| \begin{array}{l} (g \circ f)(x) = g(f(x)) \\ (h \circ g \circ f)(x) = h(g(f(x))) \end{array} \right.$$

$$\left. \begin{array}{l} f(x) = x^2 \\ g(x) = \log(x) \end{array} \right\} \rightarrow \begin{array}{l} (f \circ g)(x) \\ (g \circ f)(x) \end{array}$$

$$\left. \begin{array}{l} f(x) = \sqrt{x} \\ g(x) = \sin(x) \\ h(x) = x^3 \end{array} \right\} \rightarrow \begin{array}{l} (f \circ g \circ h)(x) \\ (g \circ h \circ f)(x) \\ (h \circ g \circ f)(x) \end{array}$$

$$\left. \begin{array}{l} f(x) = x^m \\ g(x) = \cos\left(\frac{\pi}{m}x\right) \\ h(x) = e^x \end{array} \right\} \rightarrow \begin{array}{l} (f \circ g \circ h)(x) \\ (g \circ f)(x) \\ (f \circ g)(x) \\ (h \circ g \circ f)(x) \end{array}$$

$$\left. \begin{array}{l} f(x) = x + e^x \\ g(x) = \tan(x) \\ h(x) = \sqrt{x} \end{array} \right\} \rightarrow \begin{array}{l} (f \circ g)(x) \\ (f \circ g \circ h)(x) \\ (h \circ g)(x) \\ (h \circ f \circ g)(x) \end{array}$$

2) Calcola il dominio delle seguenti funzioni.

$$\begin{array}{l} \bullet y = \sqrt{|x|} \quad \bullet y = \ln\left(\frac{1-3x}{7x+2}\right) \quad \bullet y = \frac{1}{\sin(x)} \quad \bullet y = x^3 \sqrt{x^2+3x} \\ \bullet y = e^{\sqrt{x+1}} \quad \bullet y = \ln(x^2+2x+1) \quad \bullet y = \frac{\sqrt{e^x-2}}{x} \quad \bullet y = \ln(1+e^{\frac{x}{3}}) \\ \bullet y = e^{\sqrt{\frac{e^{-x}-1}{e^{3x}-6}}} \quad \bullet y = \ln(|x-1| - \sqrt{x}) \quad \bullet y = \sin(x + 2\cos(x)) \end{array}$$

3) Studiare eventuali simmetrie delle funzioni al punto (2) (PARI, DISPARI)

4) Stabilire la natura delle seguenti funzioni. (INIETTIVE, SURIETTIVE, BIETTIVE)

$$\begin{aligned} & \bullet y = \frac{1}{2}x + 3 & \bullet y = x^2 - 2x & \bullet y = -\frac{1}{2}x - 2 & \bullet y = x^2 - 1 \\ & \bullet y = \frac{1}{2}\sqrt[3]{x} & \bullet y = \sqrt[3]{x} - 1 & \bullet y = 3x - 2 & \bullet y = \frac{x-3}{2x+1} \\ & \bullet y = \frac{2}{\sqrt{x}} & \bullet y = e^{1-2x} & \bullet y = 3^{x^2} & \bullet y = \ln(x+4) \end{aligned}$$

5) Calcolare le funzioni inverse delle funzioni biettive dell'esercizio (4)

6) Dire come poter rendere invertibili le funzioni in (4) che non lo risultano

7) Risolvere le seguenti equazioni esponenziali

$$\begin{aligned} & \bullet 10^x = 0,01 & \bullet 2^{5x} = 32 & \bullet 3^{x+2} = 81 & \bullet a^{6x} = a^{x^2}, a > 0 \\ & \bullet a^x = -2, a > 0 & \bullet \left(\frac{1}{3}\right)^{x+1} = 9 & \bullet 7^{x^2+4x+3} = \frac{1}{7} & \bullet 7^{\sqrt{x^2+1}} = 49 \\ & \bullet 2^{x+2} + 2^x = 640 \\ & \bullet 4^x + 2^x - 2 = 0 & \bullet 3^{2x} - 9 \cdot 3^x + 3 = 0 & \bullet 20^{2x+1} + 2 = 9 \cdot 20^x \\ & \bullet 3^x = 5^{x-2} & \bullet 3^{2x} - 3^{x+1} = -2 & \bullet 7^{2x-1} - 14^{2x} = 0 \end{aligned}$$

8) Risolvi le seguenti disequazioni.

$$\left(\frac{1}{5}\right)^{\sqrt{x}} > 25 \quad 16^{x^2+x} < 4 \quad \left(\frac{1}{2}\right)^x \leq -1 \quad 2^x > \frac{1}{64}$$

$$\left(\frac{2}{3}\right)^{x^2+2} - \left(\frac{27}{8}\right)^x < 0 \quad \frac{2^{3x}}{4^{x+1}} - \left(\frac{1}{2}\right)^x > 0$$

$$3^{2x} - 3^{x-1} \geq 0 \quad 2^{2x} - 3 \cdot 2^x + 2 \geq 0 \quad \left(\frac{1}{2}\right)^{2x} - 3\left(\frac{1}{2}\right)^x - 4 \geq 0$$

9) Risolvi le seguenti equazioni, logaritmiche

$$\log_3 x = 2 \quad \log_4 (2x-1) = \log_4 x \quad \ln(x+2) = 0$$

$$\log_5 (x^2+x+1) = 1 \quad \log_x 2 = \frac{1}{\sqrt{2}} \quad \ln^3 x - 9 \ln x = 0$$

$$\ln^2 x - 2 \ln x + 1 = 0 \quad 2 \ln^2(x-1) - 5 \ln(x-1) + 2 = 0$$

$$\log_3^2 (2x^2-x) = 1 \quad \log_a (x-5) + \log_a (x-7) + \log_a 3 = 0$$

10) Risolvi le seguenti disequazioni, logaritmiche

$$\ln x > 0 \quad \log_6 (x^2-x) \leq 1 \quad \ln(x+1) < 0$$

$$\ln(x^2-x) > 1 \quad \ln 5^x + \ln 5^{-2x} < 0 \quad \log_9 x \leq 10$$

$$\ln(-1+x) \geq 2 \quad \ln x + \ln 3 < \ln x^2 \quad \log_{\frac{1}{2}} x > -1$$

$$2 \log_2 x - 3 < 0 \quad 2 - 4 \log^2 x \geq 0 \quad 4 \log^2 x \geq 4$$

